Topics for a master's thesis

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In this document, you will find a selection of topics suitable for preparing your master's thesis under my supervision. Each topic includes a brief explanation and a few references. If you need more information, please feel free to contact me.

FIGURE 1. Here is the list of theses I have supervised.

1. Formanek's theorem

Let K be a field and G a group. The *zero-divisor conjecture* for group rings asserts that the group ring $K[G]$ is a domain if and only if G is torsion-free. The conjecture has been proven affirmative for several classes of groups. In 1973, Formanek proved it for supersolvable groups. References: [**14, 19**].

2. Köthe's conjecture

Köthe's conjecture is an open problem in ring theory, formulated in [**12**] by Gottfried Köthe in 1930. The conjecture can be formulated in various different ways [**13**] and has been shown to be true for various classes of rings, such as polynomial identity rings and right Noetherian rings. References: [**14, 17**].

3. The O'Nan–Scott theorem

The result is one of the most influential theorems in the theory of permutation groups. Briefly, the O'Nan–Scott theorem describes maximal subgroups of symmetric groups. Combined with the classification of finite simple groups, there are so many applications that one cannot even count them! References: [**4, 6, 15**].

4. The Abel–Ruffini theorem

The Abel–Ruffini theorem (also known as Abel's impossibility theorem) states that there is no solution in radicals to general polynomial equations of degree five or higher with arbitrary coefficients. In 1963, Vladimir Arnold discovered a topological proof of the theorem. Arnold's proof does not rely on Galois theory. Reference: [**1**].

5. McKay's conjecture for solvable groups

Counting conjectures play a fundamental role in modern representation theory of finite groups. Among these conjectures, one stands out: McKay's conjecture. John McKay formulated the problem in 1972. The conjecture is known to be true in several cases, including that of solvable groups. Reference: [**18**].

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6. The inverse Galois problem

The Inverse Galois Problem is a fundamental question in mathematics that seeks to determine whether every finite group can be realized as the Galois group of some field extension over the rational numbers. The problem was first explicitly stated by mathematicians in the late 19th and early 20th centuries, notably by David Hilbert. Despite significant progress and partial solutions, the problem remains unsolved for many groups, making it a central topic in algebra and number theory. References: [**16, 23**].

7. Three theorems of Bieberbach

Bieberbach's theorems are fundamental results in the theory of discrete groups of isometries in Euclidean space. These theorems form the basis for the classification of crystallographic groups, which are crucial in mathematics and physics. References: [**3, 5**].

8. Frobenius groups and Thompson's theorem

Suppose a finite group contains a subgroup satisfying specific properties. Using this information, what can be said about the structure of the group itself? A classical and beautiful application of character theory is provided in understanding the structure of the so-called Frobenius groups. In 1960 Thompson proved that Frobenius kernels are nilpotent groups, confirming a long-standing conjecture of Frobenius. Except for some special cases the known proofs for the theorem of Frobenius make use of character theory (or Fourier analysis). References: [**8, 9**].

9. Wielandt's automorphism tower theorem

A beautiful theorem of Wielandt states that a the automorphism tower of a finite and centerless stabilizes in finitely many steps. To prove this theorem, subnormality is a pretty crucial idea. This powerful technique is nicely presented in Isaacs book. References: [**9, 20**].

10. Groups of central type

A finite group is said to be of central type if it possesses an irreducible complex character which takes the value zero on all non-central elements. (Equivalently, the degree of this character is the square root of the index of the center.) In the paper [**7**] of Howlett and Isaacs proved that groups of central type must be solvable. This was a conjecture of Iwahori and Matsumoto [**10**].

11. The Drinfeld double of a group algebra.

The Drinfel'd double construction a sends a finite-dimensional Hopf algebra (e.g. the group algebra of a finite group) to a quasi-triangular Hopf algebra (i.e. a Hopf algebra such that its category of modules is a braided monoidal category). References: [**2, 11**].

12. The Schur cover

The Schur multiplier of a group G is the second homology group $H_2(G, \mathbb{Z})$. It was introduced for studying projective representations. Every finite group G has at least one Schur cover E. A Schur cover E have the property that every projective representation of G can be lifted to an ordinary representation of E. References: [**21, 22**].

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